EM375 MECHANICAL ENGINEERING EXPERIMENTATION

CONSTITUTIVE EQUATIONS FOR AN ELASTOMER
"RUBBERS LAB"

PURPOSE: The objective of this laboratory is to determine the relationship between force and deflection for an elastomer. The slingshot uses "rubber bands" made from surgical tubing. The material for this tubing is a synthetic latex elastomer, and we need to know its force versus deflection relationship in order to construct a mathematical model for the slingshot. You are already familiar with the common constitutive relation called "Hooke's Law," which establishes a *linear* relation between stress and strain. However, elastomers are nonlinear, and they do not follow Hooke's Law. The data reduction for this laboratory illustrates how some nonlinear relationships can be transformed into linear ones, thus enabling simple linear regression analysis to determine nonlinear material parameters.

BACKGROUND: The distance the projectile travels after leaving the slingshot is primarily a function of how much energy is imparted to it from the slingshot. The energy in the slingshot can be obtained by integrating the force-deflection or stress-strain curve for the elastic tubing as it is stretched. To do this we need a relationship that relates the stress in the tubing with its elongation or strain. There are TWO relationships that are considered in this laboratory: **The Mooney Formula**, and the **Strain Hardening Relationship**. These are two different theories that are commonly used to model the behavior of nonlinear materials.

MOONEY FORMULA: The first relationship considered is the 2-Term Mooney Formula. In this formula C_1 and C_2 are material constants we need to identify. They have units of stress.

$$s_n = 2\left[l - \frac{1}{l^2}\right]\left[C_1 + \frac{C_2}{l}\right]$$

Even though "necking" will occur, the stress, s_n will be calculated by dividing force by the original cross sectional area. The stretch ratio, l, is the ratio of stretched length divided by original length, $l = L/L_O$. To find the constants in the Mooney Formula from test data and linear regression, rewrite (transform) the Mooney Formula as:

$$\frac{s_n}{2\left[l - \frac{1}{l^2}\right]} = C_1 + \frac{C_2}{l}$$

which plots as a straight line for $\frac{s_n}{2\left[l - \frac{1}{l^2}\right]}$ versus $\frac{1}{l}$. The slope gives constant C_2 and the intercept gives constant C_1.The zero stress-zero elongation point will not be plotted. Why not?

STRAIN HARDENING RELATIONSHIP: The second relationship we consider is a strain hardening relationship of the form:

$$s_n = s_O e_n^a$$

In this equation, the constants we are trying to identify are s_O (with units of psi) and a (the dimensionless strain hardening exponent). e_n is the strain, DL/L_O. If we transform the equation by taking natural logarithms, we get:

$$\ln(s_n) = \ln(s_O) + a\ln(e_n)$$

This relationship plots as a straight line on the transformed axes. The slope gives α. The intercept gives the natural logarithm of s_O, so you have to convert the intercept to get s_O. Note that the strain parameter, e_n, is different from the elongation parameter, l, but they are related. How?

PROCEDURE: The material relationships for the tubing can be determined by collecting force and elongation data from both types of tubing (small and large size). Note that for the tubes you are testing, both relationships are only suitable for values of l up to 3.0. Therefore, when testing the samples, **DO NOT EXCEED l = 3.0.**

1. Each group has a sample of both the small and large tubing. For each sample determine the following:

Weight, length, outside diameter, inside diameter

2. Mark on the tube an unstretched reference (gage) length, L_O, somewhere between 4 and 6 inches long. After you have marked it on the tube, measure the gage length carefully. Then tie a knot in each end of the tubing and attach one end to the bench using the block and clamp.

3. Record your data in a table similar to the following example. Ensure you record the zero-weight case, plus at least eight (8) cases with increasing weight attached up to a maximum of l = 3.0
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LOAD (lbf)	LENGTH (in)	STRESS $s = F / A_O$	ELONGATION $l = L / L_O$	STRAIN $e = (L - L_O) / L_O$
0	L_O =	0.0	1.0	0.0

4. Unload the sample, and REPEAT the measurements for a second loading cycle. Other engineering tests like this one often require you to record data during both the loading and unloading phases. For this lab, record the data ONLY during the LOADING phases. With both trials you should have at least 17 data pairs for each tube.

<u>DATA REDUCTION:</u> The data reduction is to be done using Mathcad.

1. Use coordinate transformation and linear regression to determine the Mooney constants C_1 and C_2, and the Strain Hardening constants s_O and a. Remember units!

2. Create plots for each piece of tubing for each relationship (4 plots total) showing:

   a) The transformed data and linear regression overlaid with the bands for 90% confidence interval.

   b) The raw data (load vs. extension) overlaid with the theoretically regenerated curves.

<u>REPORT:</u>

1. Your report is to be written in the "Memo Report" format.

2. The report is to include *tabulated* results. State which relationship gives a better fit to the data, and include a justification of your choice.

3. The enclosures to the report are to include (as a minimum) the tabulated raw data and the results of the data reduction (plots and coefficients). Appropriate titles, headings, legends, symbols, etc are required for all tables and graphs. Graphs must be a reasonable size, and clear.

4. Hand written work in the report ***will not be accepted.***

5. Remember to cite the lab handout as a reference, and also attach it at the end of your report.

6. <u>Collaboration.</u> Group effort throughout. One report per group. Same grade for all members of the group.